The Higgs boson as a self-similar system: Towards a new solution to the hierarchy problem

M. Ahmadvand^{1, 2, *}

¹School of Particles and Accelerators, Institute Research in Fundamental Sciences (IPM), P. O. Box 19395-5531, Tehran, Iran ²School of Physics, Damghan University, Damghan 3671645667, Iran

nysics, Dumghun Oniversity, Dumghun 50/1045

(Dated: May 3, 2024)

We propose a new solution to the hierarchy (naturalness) problem, concerning quantum corrections of the Higgs mass. Assuming the Higgs boson as a system with a self-similar internal structure, we calculate its two-point function and find that the quadratic divergence is replaced by a logarithmic one in the mass corrections. It is shown that the partonic-like distribution follows the Tsallis statistics and also high energy physics experimental data for the Higgs transverse momentum distribution can be described by a self-similar statistical model.

Keywords: The hierarchy problem, Higgs boson, Self-similar systems

I. INTRODUCTION

After the discovery of the Higgs boson with a mass around 125 GeV at the Large Hadron Collider (LHC), although the last piece of the Standard Model (SM) has been found [1–3], some features of the Higgs boson are still under debate and investigation. Moreover, there are important issues, such as the mass of neutrinos, matter-antimatter asymmetry, dark matter and the hierarchy problem, which are left unanswered in the SM and should be addressed in a more fundamental theory. On the other hand, no significant deviations from the SM predictions have been observed so far at high energy collisions and the theory may be generalized to high energy scales. In this sense, in the SM, the hierarchy problem of scales between the weak and higher energy scales is more evident when considering quantum fluctuations to calculate the Higgs squared mass corrections [4]

$$\delta m_h^2 = \frac{\Lambda^2}{v^2} \left(m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2 \right) + \mathcal{O}(\log \frac{\Lambda^2}{m_h^2}), \quad (1)$$

where Λ is the cutoff of the theory, i.e., the highest accessible energy scale, $v \simeq 246 \text{ GeV}$ is the electroweak (EW) symmetry breaking scale and m_h , $m_{W,Z}$, and m_t denote the mass of Higgs, gauge boson and top quark particles. Thus, if Λ is very large, for instance as large as the Planck mass, the corrections will be extremely greater than the Higgs mass value.

Theories beyond the SM may have an additional contribution to the Higgs mass such that it is finely adjusted to cancel δm_h^2 . Supersymmetric models [5] and composite Higgs scenarios [6] are such endeavors to avoid this problem. The fine-tuning in cancellation can be measured as [6]

$$\Delta \ge \frac{\delta m_h^2}{m_h^2} \sim \left(\frac{\Lambda}{450 \,\mathrm{GeV}}\right)^2. \tag{2}$$

Therefore, in these new theories, for $\Lambda \gg \text{TeV}$, the Higgs boson mass cannot be found due to the cancellation.

In the SM, the Higgs boson is the only elementary scalar field and all other scalars are bound states of the strongly cou-

pled sector. If the Higgs is also elaborated to be a composite bound state, it can be originated from a new strongly interacting dynamics, since Quantum Chromodynamics (QCD) cannot be responsible for its construction. That is the notion established in composite Higgs models [6–8], avoiding the hierarchy problem.

In this paper, the Higgs boson is not considered as an elementary particle but a complex system with an internal structure which reveals a statistical self-similarity behavior in that constituents are similar to the main system at a different level of scale.

Self-similar objects and patterns, known as fractals, have been widely studied in various areas of science including mathematics, biology and physics, as widely found in nature (not necessarily as exact fractals) [9]. In particle physics, especially in strong interactions, self-similar systems have been used to model hadrons which are composed of hadrons [10– 12]. These models were able to describe many features of the hadronic system consistent with experimental data. Additionally, in high energy collision experiments of strong interactions, showing nearly scale invariant properties, the transverse momentum data would be well described in this context [13–16] if the Boltzmann-Gibbs statistics were replaced by the Tsallis statistics [17, 18].

In the present work, based on the non-perturbative analysis of the two-point correlation function [19, 20], incorporating the Källen–Lehmann spectral representation, we first obtain the two-point function of the Higgs with the self-similar internal structure, whose effects give rise to a modification to the field strength renormalization factor. Then, connecting the modified factor to pertubative calculations, we show that the calculation of effective one-particle irreducible (1PI) diagrams leads to a logarithmic divergence in the Higgs mass corrections.

Such a scenario can be resulted from a new QCD-like dynamics at high energies and hence not only is the prediction of the Higgs mass naturally feasible without restricting the given confining scale to be around the TeV, and without fine-tuning due to the logarithmic divergence, but also the hierarchy problem is addressed in this setup.

^{*} ahmadvand@ipm.ir

II. CORRELATION FUNCTION

We now investigate the two-point correlation function of the Higgs, in accordance with the aforementioned considerations, for the interacting theory, $\langle \Omega | \phi(x) \phi(y) | \Omega \rangle$. Using a complete set of intermediate states at some scale level,

$$\mathbf{1} = \sum_{\boldsymbol{\psi}} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} |\boldsymbol{\psi}\rangle \langle \boldsymbol{\psi}|, \qquad (3)$$

the two-point function for $x^0 > y^0$ can be written as

$$\langle \Omega | \phi(x) \phi(y) | \Omega \rangle = \sum_{\psi} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} \langle \Omega | \phi(x) | \psi \rangle \langle \psi | \phi(y) | \Omega \rangle,$$
⁽⁴⁾

where $E_p = \sqrt{\mathbf{p}^2 + m^2}$ and $|\psi\rangle$ is the self-similar partonic state which can be an eigenstate of 4-momentum P^{μ} . Because of translational invariance,

$$\langle \Omega | \phi(x) | \psi \rangle = \langle \Omega | \phi(0) | \psi \rangle e^{-ip.x} \langle \psi | \phi(y) | \Omega \rangle = \langle \psi | \phi(0) | \Omega \rangle e^{ip.y},$$
 (5)

thus Eq. (4) is expressed as¹

$$\langle \Omega | \phi(x) \phi(y) | \Omega \rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{i e^{-ip.(x-y)}}{p^2 - m^2 + i\varepsilon} |\langle \Omega | \phi(0) | \psi \rangle|^2.$$
(6)

For $y^0 > x^0$ the same procedure can be written to express the time-ordered product of the function. Furthermore, we can represent the scalar function as the Källen-Lehmann representation in terms of a spectral function

$$\langle \Omega | T\phi(x)\phi(y) | \Omega \rangle = \int_0^\infty \frac{dM^2}{(2\pi)} \rho(M^2) D_F(x-y;M^2), \quad (7)$$

where D_F is the Feynman propagator. Rewriting the effective state in terms of self-similar partonic constituent states

$$|\psi\rangle = \sum_{i} \langle \psi_i |\psi\rangle |\psi_i\rangle, \qquad (8)$$

we can explore the effect of internal structure. In this case the spectral function of the system will be

$$\rho(M^2) = \sum_{i} (2\pi)\delta(M^2 - m_h^2) |\langle \Omega | \phi(0) | \psi_i \rangle|^2 |\langle \psi_i | \psi \rangle|^2.$$
(9)

As a result, the two-point function becomes

$$\langle \Omega | T\phi(x)\phi(y) | \Omega \rangle \sim \sum_{i} \int_{0}^{\infty} dM^{2} \,\delta(M^{2} - m_{h}^{2}) Z |\langle \psi_{i} | \psi \rangle|^{2} D_{F}.$$
(10)

Note that analogous to the fractal concept and the notion of hadrons composed of hadrons, we consider the Higgs as a self-similar partonic-like state at some energy scale, $|\psi\rangle$, so that it has self-similar internal subsystems, $|\psi_i\rangle$, similar but with different energy levels with respect to the system. Thus, $Z = |\langle \Omega | \phi(0) | \psi_i \rangle|^2$ is considered as the field strength renormalization of a representative self-similar Higgs state and the effect of self-similar subsystems is included in the new probability term $\sum_i |\langle \psi_i | \psi \rangle|^2$ as

$$\sum_{i} |\langle \psi_i | \psi \rangle|^2 = \sum_{i} \left(\prod_{i} \int \frac{d^4 p_i}{(2\pi)^4} \right) |\mathcal{M}|^2 f(2\pi)^4 \delta^4 \left(\sum p_i - k \right).$$
(11)

Here, \mathcal{M} is the probability amplitude of finding the subsystem *i*th as a fraction of the system, and since ψ_i is also a self-similar partonic-like state with self-similar constituents, we consider the parameter *f* as the partonic-like distribution including the effect of substructures on the probability. Other terms in Eq. (11) denote the phase space integral over ψ_i , and *k* is the momentum of the intermediate state.²

As for calculating the probability amplitude $|\mathcal{M}|^2$, we approximate it as the fraction of the momentum of the state $|\psi\rangle$ carried on by the constituent states $|\psi_i\rangle$, i.e., $|\mathcal{M}|^2 \sim p_i^2/k^2 = m_i^2/k^2 = x^2$, where we define $x = p_i/k$ as the scale invariant parameter of the system (analogous to the parameter defined in the parton model 0 < x < 1, see, e.g., [20]).³ Then, one should find how this scale invariant parameter of the system is distributed to partons through the distribution function.

To obtain the internal distribution, we start with a noninteracting *N*-particle system whose energy distribution can be given by the Maxwell distribution

$$f(K) = \frac{\mu^{-\frac{3N}{2}}}{\Gamma(3N/2)} K^{\frac{3N}{2}-1} e^{-\frac{K}{\mu}},$$
 (12)

where *K* is the energy of the system, μ is the average energy, and Γ denotes the gamma function. However, we intend to obtain the energy distribution for a system having self-similar internal structures, which contribute to the energy of the system, and each constituent itself has self-similar subsystems. Due to this self-similar feature, we assume the attributed energy to a subsystem is a fraction of the energy of this system at different orders of the scale invariant parameter. Therefore, taking the contribution of all subsystems into account, we consider the total energy of the system as U = Kg(x), where g(x)can be generally expressed as $g(x) = 1 + ax + bx^2 + \cdots$, so that the energy of the system can be approximated as

$$U \sim K \left(1+x\right)^{\alpha},\tag{13}$$

where α denotes the fractal index and will be determined in the following by the relevant statistics of the system. As expected, the energy distribution of internal subsystems follow

¹ A sum over ψ would contribute a branch cut in the spectrum. However, since the Higgs is considered as a self-similar partonic-like state, this is not the case here.

² In the perturbative method with Feynman diagrams, k is the momentum which is integrated over at the loop level.

³ Because of the self-similarity feature of internal structures, we consider such a scale invariant parameter which remains constant at different scale levels in the system.

the main structure. Thus, for this system we can write

$$f(U) = \frac{\mu^{-\frac{3N}{2}}}{\Gamma(3N/2)} U^{\frac{3N}{2}-1} e^{-\frac{U}{\mu}}$$
$$= \frac{\mu^{-\frac{3N}{2}}}{\Gamma(3N/2)} K^{\frac{3N}{2}-1} (1+x)^{\frac{3N}{2}\alpha-\alpha} e^{-\frac{K}{\mu}(1+x)^{\alpha}}, \quad (14)$$

and hence the internal distribution would be

$$f(x) = \int dK \frac{\mu^{-\frac{3N}{2}}}{\Gamma(3N/2)} K^{\frac{3N}{2}-1} (1+x)^{\frac{3N}{2}\alpha-\alpha} e^{-\frac{K}{\mu}(1+x)^{\alpha}}$$
(15)
= $(1+x)^{-\alpha}$.

It is tempting substitute $\alpha = q/(q-1)$ and $x = (q-1)\varepsilon/\tilde{\varepsilon}$, and it can be seen the distribution, $[1 + (q-1)\varepsilon/\tilde{\varepsilon}]^{-q/(q-1)}$, follows the non-extensive Tsallis statistics, where q is the entropic factor of the system and for the limit $q \to 1$ the Boltzmann-Gibbs is reproduced.

Putting the calculated terms into Eq. (11), we obtain

$$\sum_{i} \left(\prod_{i} \int \frac{d^{4} p_{i}}{(2\pi)^{4}} \right) x^{2} (1+x)^{-\alpha} (2\pi)^{4} \delta^{4} \left(\sum p_{i} - k \right)$$
$$\sim \sum_{i} \left(\prod_{i} \int \frac{d^{4} p_{i}}{(2\pi)^{4}} \right) \frac{m_{i}^{2}}{k^{2}} \left(1 - \alpha \frac{p_{i}}{k} \right) (2\pi)^{4} \delta^{4} \left(\sum p_{i} - k \right)$$
$$\sim \sum_{i} m_{i}^{2} / k^{2} = M^{2} / k^{2}.$$
(16)

The integral can approximately be calculated by using the expanded form of $(1+x)^{-\alpha}$ as powers of x. The terms with different powers of x result in higher powers of k^2 in the denominator, and then produce finite terms in the following loop calculations; thereby we ignore these terms in this discussion. Eventually, we obtain the Fourier transformation of the two-point function as follows

$$\int d^4x \, e^{ip(x-y)} \langle \Omega | T\phi(x)\phi(y) | \Omega \rangle \sim \frac{i\widetilde{Z}}{p^2 - m_h^2 + i\varepsilon}, \quad (17)$$

where the effect of the internal structure is included in $\tilde{Z} \equiv Z\tilde{f}$ and $\tilde{f} = m_h^2/k^2$.

In the perturbative method, analogous to the sum of 1PI diagrams, denoted by \mathbb{M} , for this system the Higgs two-point function can be expressed as a geometric series of modified diagrams, $\widetilde{\mathbb{M}}$, so that

$$\frac{iZ}{p^2 - m_h^2 + i\varepsilon} \sim \frac{i}{p^2 - m_{0h}^2 - \widetilde{\mathbb{M}}}$$
(18)

where $\widetilde{\mathbb{M}}$ can be attained from Eq. (18) by expanding the denominator of the right hand side close to the pole

$$\left(p^2 - m_h^2\right) \left(1 - \frac{d\widetilde{\mathbb{M}}}{dp^2}\Big|_{p^2 = m_h^2}\right) + \mathcal{O}\left((p^2 - m_h^2)^2\right), \quad (19)$$

and comparing it to the left hand side of the equation. Thereby, from $\tilde{Z}^{-1} \sim 1 - \delta \tilde{Z}$, we can find $\delta \tilde{Z} = \tilde{f} \delta Z =$ $d\widetilde{\mathbb{M}}/dp^2 = \tilde{f}d\mathbb{M}/dp^2$ and hence $\widetilde{\mathbb{M}} = \tilde{f}\mathbb{M}$. This result can be also consistently applied to the Lehmann-Symanzik-Zimmermann (LSZ) formula [21] in that the sum of 1PI insertions in the propagator is equal to that of amputated scattering diagrams, implying $\mathcal{M}(p \to p) = \widetilde{Z}\mathbb{M} = Z\widetilde{\mathbb{M}}$.

To clarify this effect, applying the cutoff regularization, we calculate $\widetilde{\mathbb{M}}$ at the one-loop order, for the Feynman diagram with the self coupling interaction; the same procedure holds for other Higgs interactions.⁴ Thus, for the mentioned diagram

$$-i\widetilde{\mathbb{M}} = \frac{-i\lambda}{2} \int \frac{d^4k}{(2\pi)^4} \frac{m_h^2}{k^2} \frac{i}{(k^2 - m_h^2)} = -\frac{i\lambda m_h^2}{32\pi^2} \log \frac{\Lambda^2}{m_h^2}.$$
(20)

As can be seen from Eq. (20), and generally for other Higgs two-point function diagrams, we deal with the logarithmic divergence which is canceled by δm_h^2 .

III. DISCUSSION

As already mentioned, the obtained distribution obeys the non-extensive Tsallis statistics. The non-extensivity can be realized from the non-additive entropy [17] and the factor q is a measure of non-additivity.

In high energy experiments, such a distribution has been used for describing the transverse momentum p_T distribution of hadronic systems whose experimental data in a good agreement can be modeled by the following fitting function [22–26]

$$\frac{dN}{Ndp_{\rm T}} = \tilde{c}p_{\rm T}\frac{(n-1)(n-2)}{nC(nC+m(n-2))} \left(1 + \frac{\sqrt{p_{\rm T}^2 + m^2 - m}}{nC}\right)^{-n},$$
(21)

where \tilde{c} is the normalization, *n*, *C* are fitting parameters and *m* stands for the hadron mass. The number of events for a given cross section is $N = \sigma \int \mathcal{L}dt$ and $\int \mathcal{L}dt$ is the integrated luminosity. The fitting parameters can also be identified in terms of Tsallis parameters T_0 and *q* as $n \equiv q/(q-1)$ and $nC \equiv T_0/(q-1)$ [14].

In addition, in high energy collisions, the Higgs transverse momentum p_T^H is a key observable due to which one can study its properties and the dynamics of the produced system as well as distortions of its SM predictions. We try to describe the p_T distribution of the Higgs system by the mentioned statistical model, the Tsallis function, with its parameters. We consider the combined differential cross section of $H \rightarrow \gamma\gamma$ (diphoton), $H \rightarrow ZZ^*$ and $H \rightarrow b\bar{b}$ (a bottom quark-antiquark pair) decay channels for p_T^H reported by the CMS collaboration at $\sqrt{s} = 13 \text{ TeV}$ [27]. (Similar measurements reported by the ATLAS collaboration can be found in [28].) We fit the spectra for $d\sigma/dp_T^H$ to the Tsallis fit function, fixing \tilde{c} to the measured total cross section 61.1 pb [27]. As shown in Fig. (1),

⁴ Although we do not study the UV completion of the scenario here, we still assume that SM interactions hold and can be generated at low energies.

the experimental data can be well fitted to the function for $q = 1.79 \pm 0.13$ and $T_0 = 3.7 \pm 0.8$ GeV. The result can be the evidence for the self-similarity feature of the Higgs and also using fitting parameters, more detailed investigation may constrain the mass spectrum of this type of particles in future high energy physics explorations.

Another aspect of Higgs properties can be studied through the running of its interaction coupling constants by means of the beta function and the Callan-Symanzik equation. In the present setup, taking into account the modified 1PI diagrams, tiny corrections can be obtained and we leave detailed calculations associated with this feature of the model for a future work.

IV. ACKNOWLEDGMENTS

The author would like to thank Amjad Ashoorioon and Abasalt Rostami for useful comments, and Hadi Hashamipour for the help to provide the figure.

- G. Aad *et al.* [ATLAS], "Observation of a new particle in the search for the Standard Model Higgs boson with the AT-LAS detector at the LHC," Phys. Lett. B **716**, 1-29 (2012) [arXiv:1207.7214 [hep-ex]].
- [2] S. Chatrchyan *et al.* [CMS], "Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC," Phys. Lett. B **716**, 30-61 (2012) [arXiv:1207.7235 [hep-ex]].
- [3] S. Chatrchyan *et al.* [CMS], "Observation of a New Boson with Mass Near 125 GeV in *pp* Collisions at \sqrt{s} = 7 and 8 TeV," JHEP **06**, 081 (2013) [arXiv:1303.4571 [hep-ex]].
- [4] M. J. G. Veltman, "The Infrared Ultraviolet Connection," Acta Phys. Polon. B 12, 437 (1981).
- [5] P. Draper and H. Rzehak, "A Review of Higgs Mass Calculations in Supersymmetric Models," Phys. Rept. 619, 1-24 (2016) [arXiv:1601.01890 [hep-ph]].
- [6] G. Panico and A. Wulzer, "The Composite Nambu-Goldstone Higgs," Lect. Notes Phys. 913, pp.1-316 (2016) [arXiv:1506.01961 [hep-ph]].
- [7] D. B. Kaplan, H. Georgi and S. Dimopoulos, "Composite Higgs Scalars," Phys. Lett. B 136, 187-190 (1984).
- [8] M. Ahmadvand, "Matter and dark matter asymmetry from a composite Higgs model," Eur. Phys. J. C 81, no.4, 358 (2021) [arXiv:2010.10121 [hep-ph]].
- [9] B. B. Mandelbrot, "The Fractal Geometry of Nature," W. H. Freeman and Company: New York, USA (1983).
- [10] R. Hagedorn, "Statistical thermodynamics of strong interactions at high-energies," Nuovo Cim. Suppl. 3, 147-186 (1965) CERN-TH-520.
- [11] S. C. Frautschi, "Statistical bootstrap model of hadrons," Phys. Rev. D 3, 2821-2834 (1971).
- [12] G. F. Chew, "Hadron bootstrap hypothesis," Phys. Rev. D 4, 2330-2335 (1971).
- [13] I. Bediaga, E. M. F. Curado and J. M. de Miranda, "A Nonextensive thermodynamical equilibrium approach in e+ e-—> hadrons," Physica A 286, 156-163 (2000) [arXiv:hepph/9905255 [hep-ph]].

1.75MINUIT Fit CMS 1.50 1.25 $\frac{d\sigma}{dP_T^H} \left[\text{pb/GeV} \right]$ 1.00 0.750.500.250.00 100 200 300 400 500 0 P_T^H [GeV]

FIG. 1. The combined measurement of differential cross section for $H \rightarrow \gamma\gamma$, $H \rightarrow ZZ^*$ and $H \rightarrow b\bar{b}$ decay channels as a function of p_T^H is shown as black points with error bars [27]. Using the MINUIT package [29], the spectra are fitted to the Tsallis function for $q = 1.79 \pm 0.13$ and $T_0 = 3.7 \pm 0.8$ GeV.

- [14] J. Cleymans and D. Worku, "The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC," J. Phys. G **39**, 025006 (2012) [arXiv:1110.5526 [hep-ph]].
- [15] L. Marques, E. Andrade-II and A. Deppman, "Nonextensivity of hadronic systems," Phys. Rev. D 87, no.11, 114022 (2013) [arXiv:1210.1725 [hep-ph]].
- [16] A. Deppman, E. Megias and D. P. Menezes, "Fractals, nonextensive statistics, and QCD," Phys. Rev. D 101, no.3, 034019 (2020) [arXiv:1908.08799 [hep-th]].
- [17] C. Tsallis, "Possible Generalization of Boltzmann-Gibbs Statistics," J. Statist. Phys. 52, 479-487 (1988).
- [18] E. M. F. Curado and C. Tsallis, "Generalized statistical mechanics: Connection with thermodynamics," J. Phys. A 24, L69-L72 (1991) [erratum: J. Phys. A 25, 1019 (1992)] CBPF-NF-041-90.
- [19] S. Weinberg, "The Quantum theory of fields. Vol. 1: Foundations".
- [20] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory".
- [21] H. Lehmann, K. Symanzik and W. Zimmermann, "On the formulation of quantized field theories," Nuovo Cim. 1, 205-225 (1955).
- [22] B. I. Abelev *et al.* [STAR], "Strange particle production in p+p collisions at s**(1/2) = 200-GeV," Phys. Rev. C 75, 064901 (2007) [arXiv:nucl-ex/0607033 [nucl-ex]].
- [23] G. Aad *et al.* [ATLAS], "Charged-particle multiplicities in pp interactions measured with the ATLAS detector at the LHC," New J. Phys. **13**, 053033 (2011) [arXiv:1012.5104 [hep-ex]].
- [24] A. Adare *et al.* [PHENIX], "Identified charged hadron production in p + p collisions at $\sqrt{s} = 200$ and 62.4 GeV," Phys. Rev. C **83**, 064903 (2011) [arXiv:1102.0753 [nucl-ex]].
- [25] K. Aamodt *et al.* [ALICE], "Production of pions, kaons and protons in *pp* collisions at $\sqrt{s} = 900$ GeV with ALICE at the LHC," Eur. Phys. J. C **71**, 1655 (2011) [arXiv:1101.4110 [hep-ex]].

- [26] V. Khachatryan *et al.* [CMS], "Strange Particle Production in *pp* Collisions at $\sqrt{s} = 0.9$ and 7 TeV," JHEP **05**, 064 (2011) [arXiv:1102.4282 [hep-ex]].
- [27] A. M. Sirunyan *et al.* [CMS], "Measurement and interpretation of differential cross sections for Higgs boson production at $\sqrt{s} = 13$ TeV," Phys. Lett. B **792**, 369-396 (2019) [arXiv:1812.06504 [hep-ex]].
- [28] M. Aaboud *et al.* [ATLAS], "Combined measurement of differential and total cross sections in the $H \rightarrow \gamma\gamma$ and the $H \rightarrow$

 $ZZ^* \rightarrow 4\ell$ decay channels at $\sqrt{s} = 13$ TeV with the ATLAS detector," Phys. Lett. B **786**, 114-133 (2018) [arXiv:1805.10197 [hep-ex]].

[29] F. James, "MINUIT Function Minimization and Error Analysis: Reference Manual Version 94.1," CERN-D-506.